Measuring Boltzmann's Constant by Analysis of Johnson Noise in Resistors

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Johnson Noise from a variety of resistors was measured and analyzed. Measurements of the Boltzmann constant (K_b) were made by computing the power spectrum of each resistor's Johnson Noise, along with its temperature and resistance. The mean K_b value measured for 8 resistors ranging in resistance from 15000 omhs to 390000 omhs was 5.4110^{-23}(SD = 6.9275910^{-23}), which is within 2.92% of the literature value of $K_b = 1.38064910^{-23}$[3]. Results from our experiment support the theory of Johnson Noise.

Johnson Noise (also known as Thermal noise) is defined as the fluctuating voltage in a conductor or semiconductor caused by the random motion of charge carriers within the material [1]. This is due to the charge carriers within the material being in a state of thermal agitation, necessary to maintain thermodynamic equilibrium with motion of atoms in the conductor due to non-zero heat[2]. Johnson Noise is small in magnitude but non-trivial for many applications such as in radio transmitting and receiving[3].

The equation for Johnson Noise is

$$4TK_bR = (\frac{V^2}{\Delta f})$$

where $\frac{V^2}{\Delta f}$ is the power spectrum, T is the temperature of the material and K_b is the Boltzmann constant, $K_b = 1.38064910^{23}$[4]. In this letter, we will present measured values of the K_b constant determined using a power spectrum analysis of the Johnson Noise in common resistors.

The voltage fluctuations were found to be in the magnitude of $10^{-15}V$ to $10^{-17}V$. In order to measure a signal of such small amplitude, we used an amplifier to boost the signal to a range which could be picked up by a consumer laptop-interfacing soundcard. The circuit is shown in figure 1. Johnson noise was recorded using the audio recording software Audacity on an Apple laptop. Measurements were exported as .wav files and further analysis was performed using Python. Fourier analysis was implemented to compute the power spectrum of each sample, which was used along with temperature and resistance measurements to make an empirical measurement of K_b in accordance with the equation above.

In order to obtain meaningful quantitative data from the system, a conversion function of the soundcard output to voltage was determined. A known low amplitude sinusoidal signal was provided by a function generator and a voltage divider circuit, and this signal's amplitude as measured by Audacity was compared with measurements made by oscilloscope (in Volts). We derived a fit function used to convert the unit-less Audacity data recorded by the soundcard into Volts. The conversion from Audacity data to Volts was observed to be linear as anticipated. The fit function that was incorporated into the analysis code was $y = 5.3052x + 0.0036$ (with voltage being on the x-axis, Audible measurement on the y-axis).

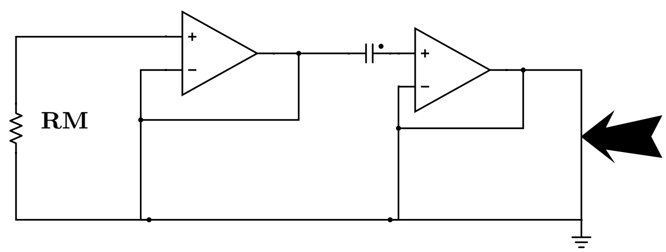

FIG. 1. Johnson Noise measurement circuit. The resistor to be measured (RM) is connected to to a BNC head and inserted into a copper sheathing capsule. The BNC connector is run to the preamp circuit which uses 2 OP-amps in series separated by a 0.22 uF capacitor. The preamp is powered by 4 D batteries. The black arrow indicates the position of the soundcard used to record the noise onto a laptop.

The pre-amp was tested in order to characterize the gain it provided. Due to the soundcard's limitation on signal amplitude that it can measure without clipping, a voltage divider circuit was used to reduce a signal provided by the function generator. Gain was measured as the V_{out} measured after the amplifier divided by the V_{in} before amplification. We found the functional range of the preamp to lie between 30 Hz and 9 kHz, outside of this range there was a steep decline in gain.

We made measurements for K_b by analyzing the power spectrum of Johnson noise in the frequency range from 550 Hz to 1700 Hz. This range was selected because of the preamp's frequency response. By selecting a range in which the preamp works optimally, we can make a more accurate measurement. We approximated the gain over this range to be the constant function G = 2370 in our analysis by averaging the data points collected for the frequency response in the range.

To make a measurement for K_b for a given resistor, we analyzed the .wav file recorded using the soundcard. Because Johnson noise is random, the length of a sample is irrelevant to its power-spectrum, and thus it is not required for each sample recording to be of the same length. Our samples were approximately 2 seconds on average.

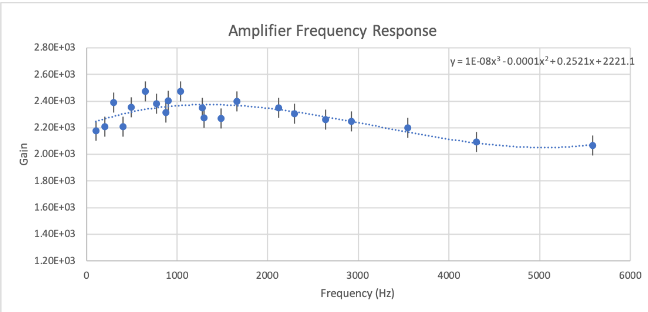

FIG. 2. Preamp Gain as a function of frequency for the range of 100 Hz to 5500 Hz. The plot is fitted with a 3rd degree polynomial function using excel. The fit equation displayed is $y = 10^{-8}x^3 - 0.0001x^2 + 0.2521x + 2221.1$. The error bars indicate the uncertainty in gain carried over from the uncertainty of the oscilloscope, which was taken to be plus or minus 6mV.

Their power spectrums were computed by fourier analysis implemented using the Scipy.signal library in Python.

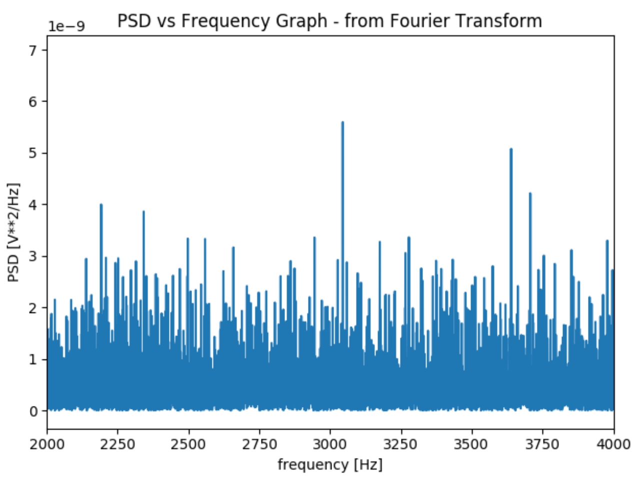

FIG. 3. Johnson Noise Power Spectrum for a 6788 Ohm resistor.

By rearranging the Johnson noise formula in terms of K_b;

$$K_b = (\frac{1}{4TR})(\frac{V^2}{\Delta f}),$$

we made measurements of K_b by taking $\frac{V^2}{\Delta f}$ to be the average of the power spectrum over the aforementioned range of frequencies, T to be the room temperature (in Kelvin) measured by digital thermometer, and R to be the resistance of the resistor measured by Digital Multimeter.

The results of these measurements varied depending on the value of the resistor. For small resistor values, such as the 200 ohm resistor, the measured K_b was not close to the literature value. This can be explained by the fact that the power spectrum of Johnson Noise is proportional to the resistance of the material, and thus for lower resistance materials the noise generated from the measurement circuit will be larger in relation. This noise is expected to come primarily from the preamp circuit, and in a lesser degree from the preamp's power supply. Other resistors generally gave values of K_b within the same order of magnitude as the literature value. See Figure 4 for the full table of results. The average K_b value from our experiment is calculated to be 5.4110^{-23}(SD $= 6.9275910^{-23}$) , which is within 2.92% of the literature value of $K_b = 1.38064910^{-23}$[3].

The results of our experiment provide evidence in support of existing Johnson Noise theory. The experiment could be improved by using two or more pre-amp circuits simultaneously to record the Johnson Noise of a resistor onto separate channels of Audacity (or a similar recording software). In this way the noise from the pre-amps themselves could be identified and accounted for during analysis, allowing for the experiment to achieve meaningful results for a wider range of resistors, particularly for small resistances. It would be of interest to use this method to perform the experiment on more resistors and examine in greater detail the relationship between the resistance and the magnitude of voltage fluctuations for a resistor.

Resistance (Ω)	Power Spectrum Average (V²/Hz)	Calculated Boltzmann constant (J/K)
201.8	2.50E-17	1.05E-22
1790	3.66E-17	1.74E-23
3290	5.74E-17	1.48E-23
6788	1.11E-16	1.39E-23
14870	2.39E-16	1.36E-23
30000	4.53E-16	1.28E-23
108000	1.61E-15	1.79E-22
179000	2.72E-15	1.29E-23
270000	4.58E-15	2.01E-22
301000	4.36E-15	1.23E-23
390000	5.79E-15	1.26E-23

FIG. 4. Table showing results for K_b for the tested resistors. The mean value of the calculated K_b is 5.4110^{-23}(SD $= 6.9275910^{-23}$). The literature value of Boltzmann's constant is $K_b = 1.38064910^{23}$[3]. Note that the measured room temperature was 294.05 K for all trials with the exception of the 201 ohm resistor, for which the measured room temperature was 295.75 K.

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