PET: a study of system geometry and image reconstruction quality

Levi Keay

August 11, 2022

Abstract

PET data quality relies on appropriate selection of scan parameters, namely the geometries of the apparatus including the aperture of the detectors. Metrics for blurring and SNR are measured in a lab setting using a 2 detector set up with 2 degrees of freedom and an Na-22 radiation source. Scans using wider apertures are confirmed to be characterized by increased blurring and higher SNR.

1 Introduction

PET is a medical imaging modality which functions on the the principle of colinear gamma ray emissions from a radiation source undergoing beta positive decay. By knowing the relative locations of the detector pairs which collect such emissions, an image of the radiation can be reconstructed as long as sufficient projections are obtained from a variety of angles. Important attributes of signal quality include spatial resolution, blurring, and signal to noise ratio (SNR). Spatial resolution can be improved by taking more projections and using finer spatial increments when scanning. Blurring can be improved by decreasing the available detector area by the narrowing of a blocking aperture, but this comes at the expense of decreasing overall count rate and thereby diminishing the SNR. All three of these attributes are of importance for accurate diagnosis and monitoring in a medical setting. While enhancing spatial resolution in the sense of pixel dimension is largely a problem of increasing the number projections by using more detectors or a more granular step sizes in a 2 detector system, the trade-off existing between blurring and SNR when it comes to aperture choice is non-trivial. In this letter I will present a method for measuring the blurring and SNR of a two detector PET system present in scans of an Na-22 radiation source in a lab setting, and provide insight into choosing an appropriate aperture value depending on use case.

The analytical model of the relative count number as a function of distance, $|x|$, is found as follows. We assume isotropic random emission from the source, and simplify the geometry to the 2 dimensional case, as shown schematically in figure 1. The relative number of photons counted as a function of the distance from the center of the imaging area, x, is given by the normalized sum of the regime areas multiplied by their respective probabilities of detecting both co-linear photons.

This can be formulated by comparing areas on the unit circle, and because the regimes are symmetrical about both the horizontal and vertical axes, we can restrain the problem to the area of one quadrant of the circle.

For the angles θ_{green}, θ_{yellow}, θ_{red} with respective probabilities of coincident detection of 1, P_{copper}, and ${P_{copper}}^2$ respectively, then the number of counts as a function of the distance from center, x, is given:

$$N(x) = \frac{\theta_{green} \cdot 1 + \theta_{yellow} \cdot P_c + \theta_{red} \cdot {P_c}^2}{\frac{\pi}{2}}$$

Where:

$$\theta_{green} = \arctan\left(\frac{\frac{a}{2} - |x|}{L}\right)$$

$$\theta_{yellow} = \arctan\left(\frac{\frac{a}{2} + |x|}{L}\right) - \theta_{green}$$

$$\theta_{red} = \arctan\left(\frac{D - |x|}{L}\right)$$

for aperture width a, source to aperture distance L, and scintillator width D. The photons counted in the P = 1 regime (green) will compose the primary signal from which information meaningful for image reconstruction can be extracted, while the yellow and red regimes will contribute to background noise. The ratio of the primary signal to the background noise signal gives the signal to noise ratio for a scan.

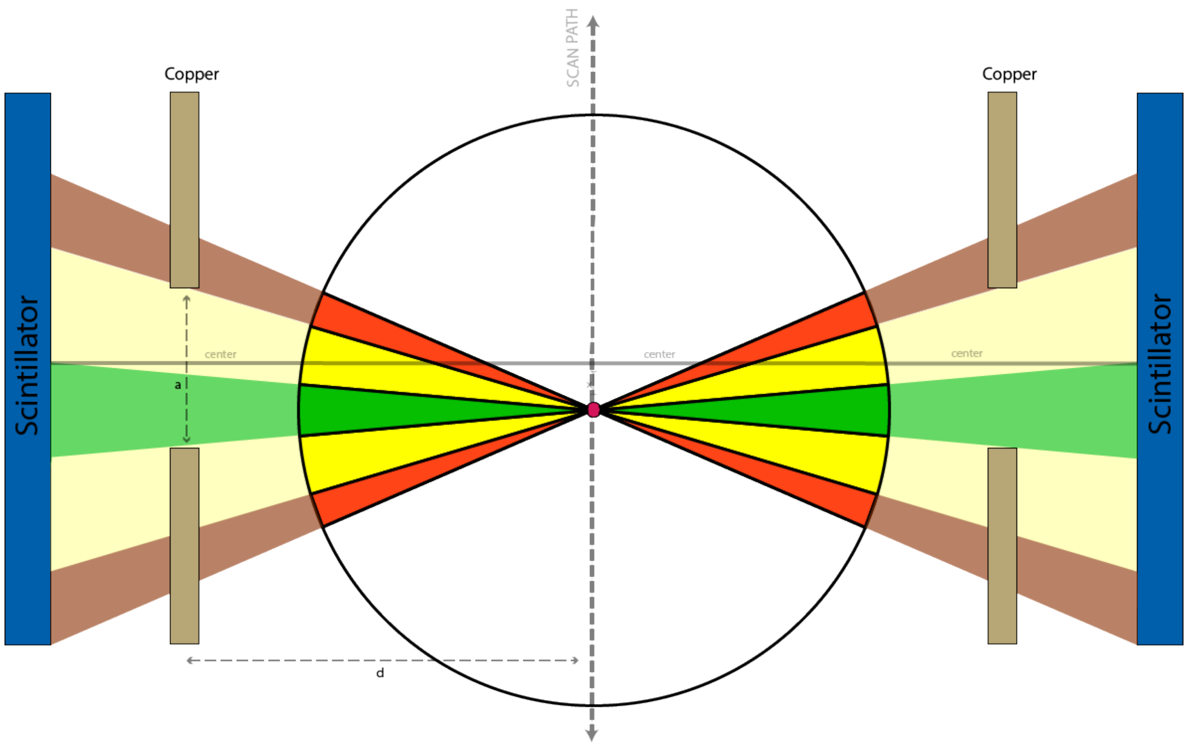

Figure 1: The possible paths of photon emission from an anisotropic radioactive source resulting in coincident scintillator activation are shown. Green areas represent lines of response in which both linearly opposing photons pass through the open aperture of the devices with probability ${P_{open}}^2 = 1$, while the red regions represent the lines of response in which both photons pass through the semi transparent copper shielding with probability ${P_{copper}}^2$. When the source is not directly in the center of the imaging area, a third regime occurs in which one photon passes through the open aperture while the other penetrates the copper with the multplied probability $P_{open} \cdot P_{copper} = P_{copper}$. The theoretical coincidence count rate as a function of the source position is determined by the sum of the areas on a unit circle multiplied by their corresponding probabilities of coincidence detection.

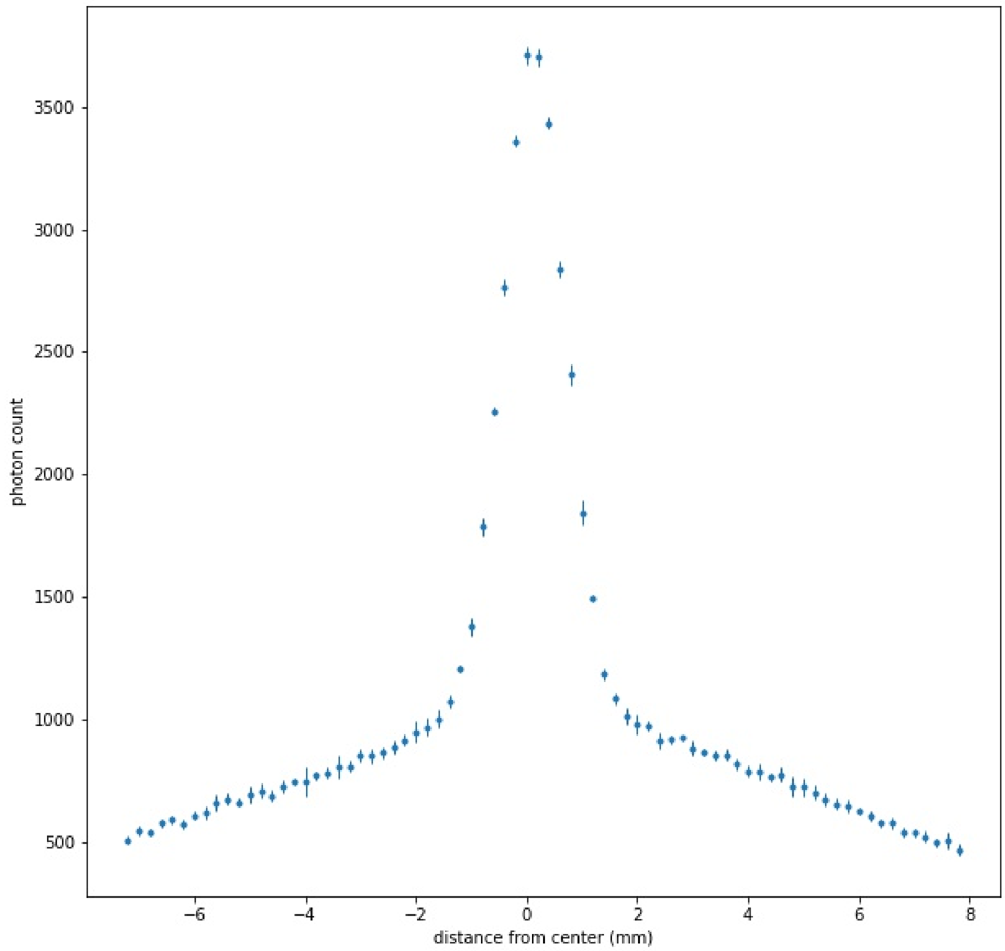

Figure 2: Coincident photon counts from a single Na-22 radiation source measured for varying distance from the center of the imaging apparatus perpendicular to the axis of the detector pair. The distance range of 15mm was covered in increments of 0.2mm and the source was held stationary for 100 seconds at each increment. The mean of 5 trials is plotted with error bars representing one standard deviation from the mean for each position. The aperture for this run was 3mm, which is distinguishable in the data as the full width of the primary signal (ie, the steeply sloped regime between -1.5mm and 1.5mm). The primary signal contains information beneficial to image reconstruction, while the tails outside of it contribute to the background noise. The characterization of the noise signal and its use in isolating the primary signal is crucial to the measurement of the signal-to-noise ratio (SNR) of the scanning system.

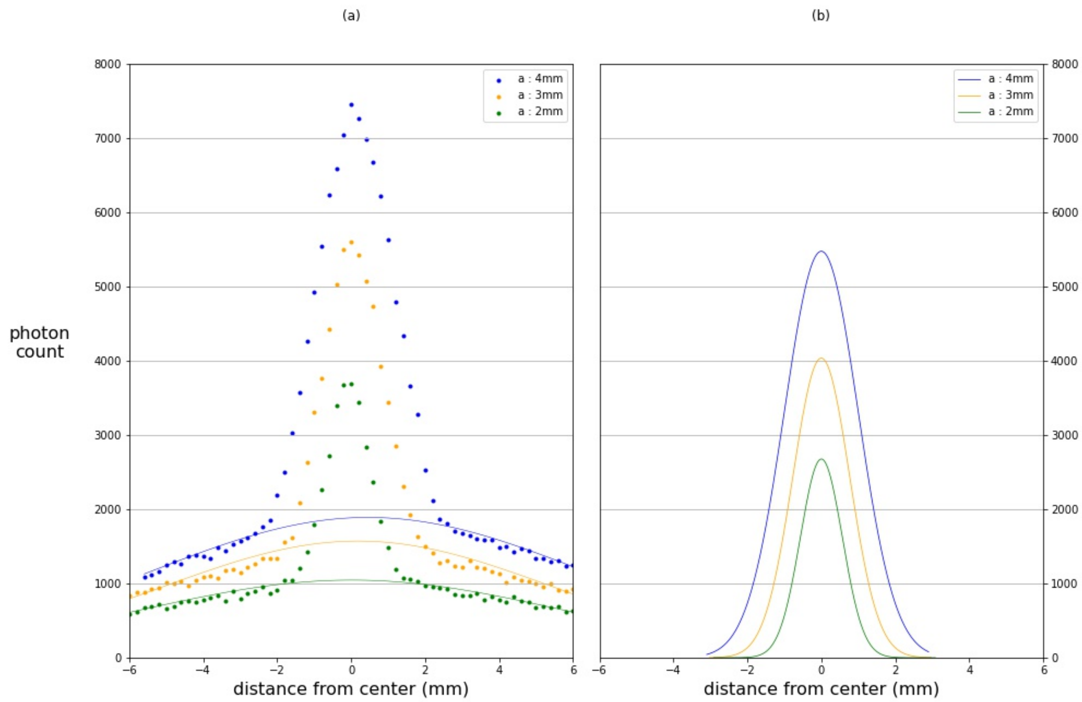

Figure 3: Coincident co-linear photon count data generated from an Na-22 radiation source is shown for three scans using varying aperture widths a. The noise signal is interpolated as a Gaussian fit to the outer tails, shown as the solid lines in (a). The isolated primary signals are fit using a separate Gaussian function (b). The primary signals are separated by first segmenting the data using the bottom-up break-point detection algorithm applied to the first spatial derivative, which then allows for the noise-signal Gaussian function to be fit to the tails of the data. This serves as an interpolation of the noise signal within the primary signal range. The primary signal is then isolated by taking the region of the data lying between the tail segments and subtracting the noise-signal Gaussian function. As expected, the distributions from trials with wider apertures are less localized and larger in amplitude due to a larger detector area being available to the source as it moves through the imaging plane.

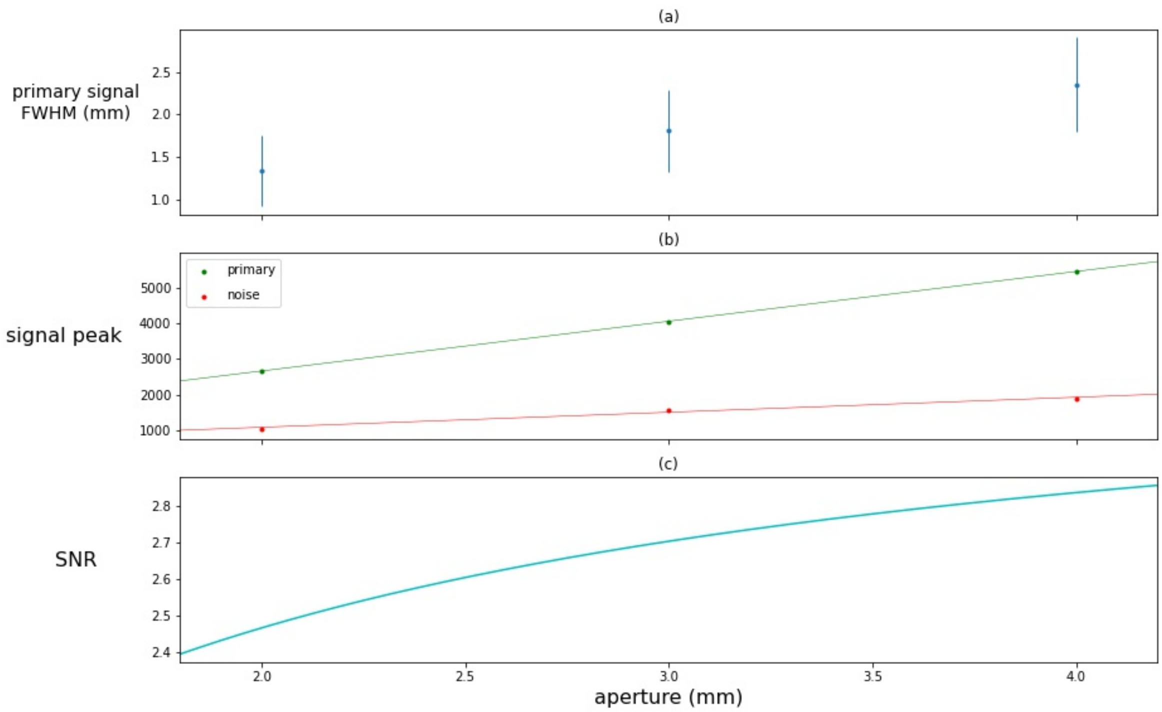

Figure 4: Signal attributes relevant for image reconstruction quality are shown as a function of aperture as determined by experiment using an Na-22 radiation source scanned in the linear horizontal plane perpendicular to a scintillator detector pair. Full-width-at-half-maximum (FWHM) of the primary signal (a) is a measure of blurring, while the relative strength of the primary and noise signals (b) give signal-to-noise ratio (SNR) (c) which determines the contrast of the radiation source to it's back ground in the reconstructed image. Error bars in (a) represent one standard deviation in the standard deviation of the Gaussian fitting function. Error in the calculation of signal peaks are carried through from error in the scaling factor of the fit of Gaussian functions to the primary signals, and they too small to be shown in the plots. When selecting an aperture, a trade-off exists in image quality between blurring and SNR. The selection of an appropriate aperture value depends on the spatial dimensions of the target, the activity and localization of the radiation source, and the spatial resolution of the scan as determined by the translatory and rotational step sizes.

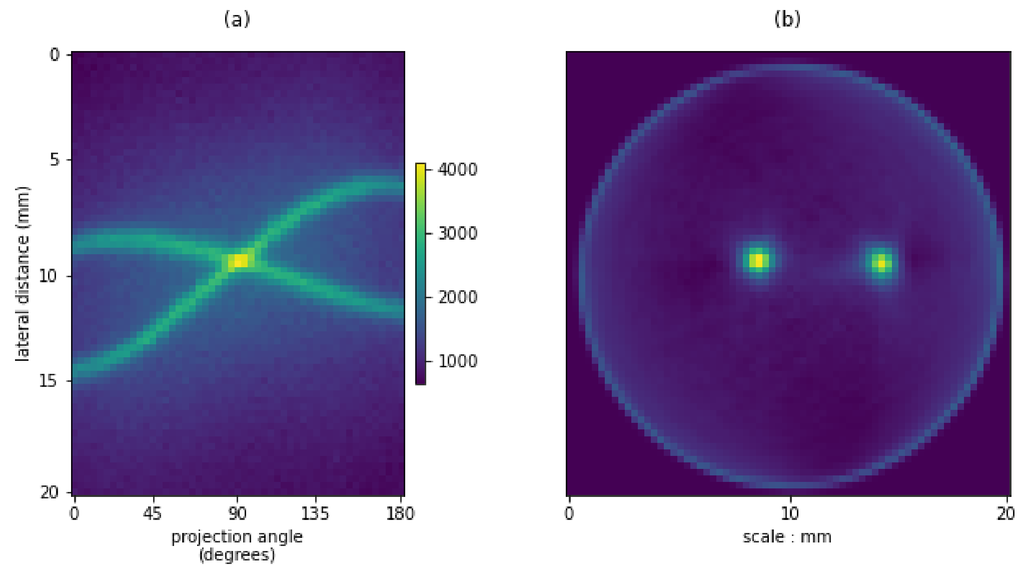

Figure 5: An example of a sinogram (a) and the reconstructed image by inverse radon transform (b) for a PET scan of two side-by-side Na-22 radiation sources. The exact size and shape of the sources is unknown but estimated to be roughly spherical diameters of 2-5 mm. The scan covered 180 degrees of rotation at a step angle of 3.6 degrees, giving a total of 50 projections. For each projection, the source arm was moved laterally through the imaging area, covering a total distance of 23mm with a step size of 0.4mm, and being held stationary for 100 seconds at each step. Each column of the sinogram (a) represents the projection obtained from one imaging angle. The reconstructed image is well resolved as the two sources are distinguishable and the region of activity is constrained to a few mm, which is comparable to the physical size of the radiation sources (b). The data was manually aligned such that the tomography rotation axis aligned with the midpoint of the sinogram.