AC susceptometry measurement and characterization of the London penetration depth in $YBa_2Cu_3O_{7-\delta}$.

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(Dated: November 26, 2021)

Advancements in the understanding and application of superconductors require the experimental characterization of key parameters in their temperature dependent behaviour. One such parameter is the London penetration depth; the depth at which a superconductor in an external magnetic field experiences a reduction in magnetic field strength by a factor of e due to the low-resistance currents induced on the surface of the conductor. In our experiment, the London penetration depth of a sample of superconducting $YBa_2Cu_3O_{7-\delta}$ (YCBO) is measured across a range of temperatures from 86 K - 106 K by AC susceptometry. The critical temperature at which the superconducting phase transition occurs and the penetration depth becomes non-zero is observed to be ($T_c = 95.7K$). Below T_c the penetration depth is observed to follow a power law, being proportional to n^{th} power of the ratio of the difference between T_c and the temperature of the sample and T_c. A fit of the measurements taken suggests a power of $n = 0.07$ (SD = 7.7×10^{-6}) in the regime of temperatures below the phase transition.

Super conductors continue to revolutionize the technology around us today, with implementations in many fields including producing strong magnetic fields in MRI machines and in satellite components to attain superior image quality. High T_c superconductors are promising for their ability to provide the advantages of superconducting components but at more easily attainable temperatures, thus broadening the scope of available implementations and decreasing costs to do so. The experimental characterization of such high T_c superconductors is paramount to further progress in the field. Significant research effort is put into the characterization of the magnetization response of super conductors [3], the magnetic relaxation [5], and the London penetration depth [4]. This research is needed in order to classify different superconducting materials and to understand their complex temperature responses. In this research we present a characterization of the magnetization temperature response of $YBa_2Cu_3O_{7-\delta}$ (YCBO) due to the Meissner effect by the method of AC susceptometry.

When a sample of superconducting matter experiences an external magnetic field, its ultra-low resistance allows for induced current vortexes to form on its surface. The currents form as to create an internal magnetization opposing the external field, and therefor resulting in a diminished magnetic field within the sample. This phenomenon is known as the Meissner effect [2][6]. The magnetic field strength inside the sample decays spatially due to the Meissner effect like

$$B = \mu_0 H_{ext} e^{\frac{-z}{\lambda_L}} \qquad (1)$$

where the z is the depth from the surface of the superconductor, λ_L is the temperature dependent London penetration depth, H_{ext} is the magnitude of the external magnetic field, and μ_0 is the constant magnetic vacuum permeability. The temperature dependent London penetration depth is of particular interest for the characterization of the Meissner effect in super conductors as it provides insight into the strength a magnetic field within a superconductor. The London penetration depth is defined as the depth into the superconductor at which the magnetic field strength is diminished by a factor of e as compared to outside the superconductor due to the Meissner effect.

Measurements of the London penetration depth can be made by AC susceptometry in which the disturbance of a uniform magnetic field due to the Meissner effect is measured. This is accomplished by the comparison of magnetic field strength between two evenly sized regions of space in a near-uniform magnetic field, with one region containing a superconducting sample while the other is empty. This is the premise of AC susceptometry and is often analyzed by an arrangement of solenoids and the use of a lock-in amplifier. Figure 1 shows a schematic diagram of the AC susceptometery apparatus used in our experiment. The signal detected by the lock-in amplifier will be:

$$V_{lock-in} = |V_{coil1} - V_{coil2}| = C(1 - \frac{2\lambda_L}{x}) \qquad (2)$$

where λ_L is the London penetration depth, x is the width of the sample tangential to the magnetic field, and C is an unknown constant which can be determined by a fit of the measurements and a known point of the penetration depth - temperature curve. The measured lock-in voltage described by (2) is a direct result of the magnetization of the YCBO sample.

Magnetization of the YCBO sample is measured by using an AC susceptometer probe consisting of two counter wound coils. An outer "drive" coil is used to provide an approximately uniform magnetic field, into which the probe is inserted 1. Inside the probe, one coil contains the sample while the other is empty. Also inside the probe is a silicon diode connected to a multi-meter in four-wire measurement mode, used to obtain precise temperature readings. Four-wire mode is required at this stage to extract a signal from the diode's temperature varying resistance of significant signal-to-noise ratio to provide a reliable temperature measurement. A heating chip resistor acts as a heating element when a small current is

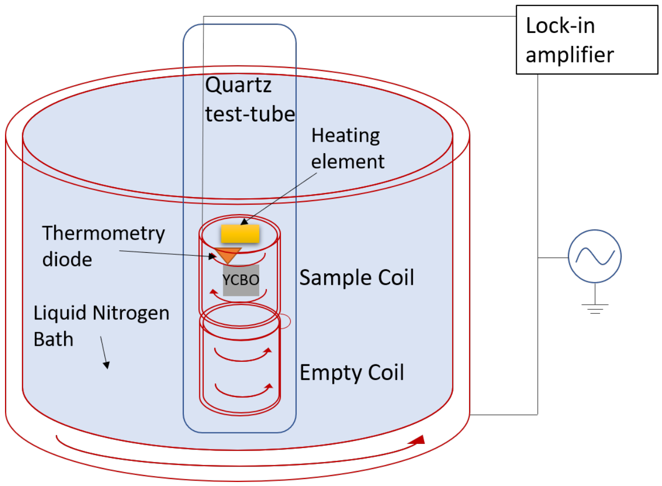

FIG. 1. Diagram of the AC susceptometer apparatus used to measure the magnetization of a sample of superconducting $YBa_2Cu_3O_{7-\delta}$(YCBO). The outer drive coil is driven by a function generator at 10V PP and 1 kHz AC signal. Two counter wound coils are connected in series; the sample coil contains the YCBO, and the other is empty. The coils are protected in a quarts test tube which is lowered to the midpoint of the drive coil, and is fully immersed in liquid nitrogen. The magnetic field produced by the drive coil induces equally opposing currents in the two inner coils when the sample is not exhibiting superconducting behaviour. When the sample is below Tc, the Meissner effect within the sample disturbs the magnetic field, and the sample coil sees a reduced field. The lock-in amplifier reads a non-zero voltage, increasing for increasing Meissner effect. The relative locations of these components are illustrated and labelled. Not shown (for simplicity) are the helium gas and vacuum pump connection, and wire connections for the heating element and thermometry diode.

run through it, and this configuration allows for the controlled and gradual temperature variation required for this experiment. A lock-in amplifier is used to measure the inductive response of the counterwound coils which are connected in series. A function generator provides a 1 kHZ signal at 10 V peak-to-peak, and the sync output of the function generator is used as the reference signal fed to the lock-in amplifier. The lock-in amplifier takes the signal induced in the probe coils as the sample signal.

When the sample is in a conducting (non-superconducting) state, the field lines penetrate it fully and the magnitude of the magnetic field contained by both coils is theoretically the same, and therefor their induced currents cancel. No signal is expected to be observed by the lock-in amplifier. When the sample is cooled below Tc and undergoes a phase transition to its superconducting state, it begins to expel the magnetic field via the Meissner effect. This results in a lower magnitude of magnetic field within the sample coil, and a difference in current between coils which is detected by the lock-in amplifier as a non-zero DC voltage. The voltage difference is proportional to the magnetization of the sample and can therefor be used to derive the London penetration depth using equation (2). Because the precise external field is not known, we normalize the measurements taken and use a literature value for YCBO's London penetration depth at a given point of $\lambda_L(77K) = 3000\mathring{A}$ to scale the magnetization measurement data and achieve a measurement of penetration depth. To fit the data, we assume a power-law relation of

$$\lambda_L(T) \propto (\frac{T_c - T}{T_c})^n \tag{3}$$

where T_c is the critical temperature and T is the temperature of the sample. n is a free parameter and is determined by the fitting process.

In order to make observations of the magnetization via the lock-in at varying temperatures, the driving coil is fitted with an inner container which can hold liquid nitrogen with the probe submerged inside. Before collecting data, the probe is centered in the drive coil by varying it's depth and observing the lock-in readout, considering it centered when the read-out voltage is minimized. The quartz test tube containing the probe is flushed with helium twice, to prevent any freezing of components due to water vapour. The test tube is then evacuated using the pump to provide improved thermal isolation, and thereby improved temperature control capabilities. Liquid nitrogen can then be poured into the drive-coil container, and the set-up allowed to cool until it reaches a temperature of 86 K. Theoretically, cooling to the temperature of the liquid nitrogen of 77 K is attainable, however this was not practical given lab hours as the apparatus can not be left unsupervised. The probe is slowly heated by applying current to the chip resistor. A computer programmed with LabView software was used to take measurements at 0.5 Kelvin intervals as the sample was heated over the range of 86K – 106K. The voltage from the lock-in amplifier (magnetization) and the temperature from the four-wire resistance measurement of the thermometry diode were recorded at each temperature step, as shown in figure 2.

While the magnetization due to the Meissner effect is of primary interest in this research, there are other temperature dependent magnetization effects present, namely the paramagnetic and diamagnetic responses [3]. In order to adjust for these effects, a linear function is fit to the magnetization – temperature data in the approximately linear regime above T_c, and linear relations ship is then subtracted from the entire dataset. In the regime above T_c, the Meissner effect is not present and therefore we are confident that a fit to this regime provides a good basis for correction of the paramagnetic and diamagnetic responses of the sample.

With the magnetization corrected to remove the spin component, we go about obtaining the London penetration depth as follows. The magnetization is proportional to the voltage measured by the lock-in amplifier. By this proportionality, we use formula (1) to extract the London penetration depth from this measurement. The magnetization data is normalized to vary between 0 and 1,

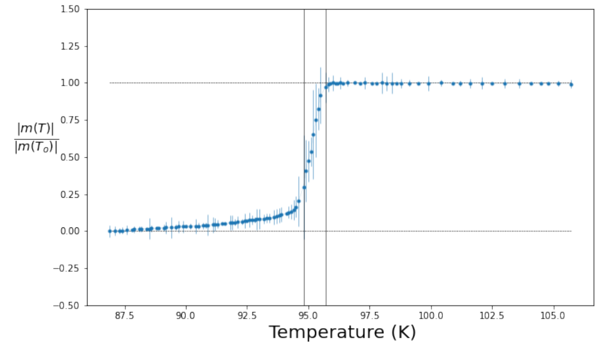

FIG. 2. Measurements of the normalized magnetization of a superconducting YCBO sample for a temperature range spanning the phase transition at $T_c = 95.7$ K are shown. The measurements have been normalized to range between zero and one (dashed horizontal lines). Spin magnetization effects have been accounted for by fitting a corrective linear function to the measurements above T_c, and subtracting this from the entire set of measurements. The phase transition is visible as the steeply sloped region. Breakpoints are denoted with vertical lines and were determined using the Kernalized Breakpoint Detection model [1] applied to the first derivative of the magnetization with respect to K. The breakpoint at 95.7 K is taken to be T_c as it is the point where the magnetization first deviates from its conducting value. The breakpoint at 94.8 K is used to separate the phase transition from the fully superconducting regime,used for fitting to extract results (see fig 3) Error bars represent standard deviation from the mean of 5 measurements taken for each temperature step and are scaled proportionally to the magnetization.

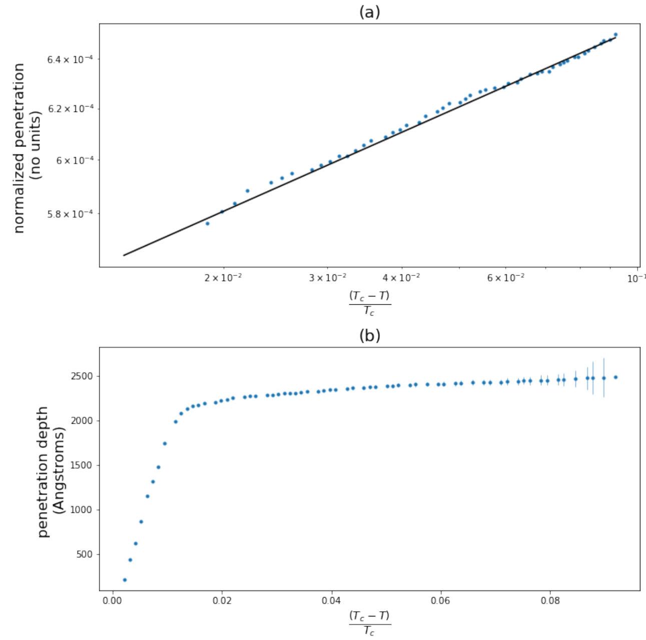

FIG. 3. Panel (a) shows a log-log plot of the London penetration depth in superconducting $YBa_2Cu_3O_{7-\delta}$ (YCBO) measured by AC susceptometry. The power-law function is shown as a straight line of slope $n = 0.072$ with estimated uncertainty for the n parameter (one standard deviation of the fit parameter) being 7.7×10^{-6}. This is obtained by fitting a linear function in the logarithmic domain to the region below the knee of the curve. Panel (b) shows the scaled penetration depth. The penetration axis is scaled by a factor 3.8×10^6 which is determined by subtracting the fit function value from the literature value for the London penetration of YCBO at liquid nitrogen temperature, 77K. Error bars are carried through and scaled appropriately from the measurements of magnetization as shown in figure 2, being the standard deviation from the mean of 5 measurements taken at each temperature interval.

thereby giving a magnetization scaled to $\frac{|m|}{|m_0|}$ where m_0 is the magnetization observed under regular conducting conditions as found at temperatures above T_c. This is as shown In figure 2. The phase transition can be seen to occur near 95.7K as a steep drop off over the temperature range between 94.8 - 95.7 K.

In the superconducting regime below T_c the YCBO has a non-zero London penetration depth λ_L. We wish to examine the relationship between this depth and temperature. There is an expected power law as expressed in formula (3). The super conducting regime $(T < 94.8K)$ was fitted to such a function, the value of the power n was determined to be 0.072 $(\text{SD} = 7.7 \times 10^{-6})$. The resulting fit of this function is shown in figure 3 as a straight line in the top panel log-scale plot. As seen by the reported SD error, the function fits the data well in the targeted region below the phase transition where the assumption $\lambda_L << x$ holds true. The fit function is then used to obtain a scale factor by comparing the normalized model value to the literature value of $\lambda_L(77K) = 3000\mathring{A}$. This allows for the scaling of the entire dataset to give the resulting bottom panel plot in figure 3, showing the London penetration depth below T_c.

The findings of this experiment support previous findings of the power-law relation of the penetration depth with temperature decreasing from T_c as described by equation (2), with the value of n found to be 0.072 with an estimated uncertainty of 7.7×10^{-6}. The uncertainty is calculated as one standard deviation in the fit of the parameter n.

No error is reported on the determined value of the T_c result because our data is not intended to be sufficiently dense in the temperature domain to make a significantly accurate assessment of this parameter. The method we employ to choose T_c using a derivative breakpoint is convenient because it can automatically identify a transition point in the data, however the sparsity of data points and the lack of physical meaning behind the point determined by this method does not allow us to contribute a result to the characterization of T_c at this time. $T_c = 95.7$ K is considered an estimation and is used as a reference point for the rest of the analysis.

From equation (1), a small value of n mathematically points to λ_L changing with temperature abruptly near the critical temperature and exhibiting fairly stable behaviour at temperature more than a few Kelvin below T_c.

This has implications on designing systems using superconductors, as we see that the majority of the magnetic effects occur close to T_c, and therefor for many applications there may be little added benefit to achieving temperatures far below T_c, as the magnetic properties do not change as substantially once in the regime of a few degrees below T_c.

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